

Confidence Intervals One Sample Mean Free Response

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Question 1

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Conditions for Interval , One Sample T Interval For A Mean

Paper: Part-A / Series: 2000 / Difficulty: Medium / Question Number: 2

2. Anthropologists have discovered a prehistoric cave dwelling that contains a large number of adult human footprints. To study the size of the adults who used the cave dwelling, they randomly selected 20 of the footprints from the population of all footprints in the cave and measured the length of those footprints. Some statistics resulting from this random sample are as follows.

Sample size	20	Minimum	15.2 cm
Mean	24.8 cm	First quartile	18.7 cm
Standard deviation	7.5 cm	Median	21.5 cm
		Third quartile	30.0 cm
		Maximum	37.0 cm

The anthropologists would like to construct a 95 percent confidence interval for the mean foot length of the adults who used the cave dwelling.

- (a) What assumptions are necessary in order for this confidence interval to be appropriate?

- (b) Discuss whether each of the assumptions listed in your response to (a) appears to be satisfied in this situation.

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Question 2

Qualification: AP Statistics

Areas: Confidence Intervals, Hypothesis Testing

Subtopics: Hypothesis Testing - One Tailed, One Sample T Interval For A Mean, One Sided Confidence Interval, One Sample T Test for a Mean

Paper: Part-B / Series: 2004 / Difficulty: Hard / Question Number: 6

6. A pharmaceutical company has developed a new drug to reduce cholesterol. A regulatory agency will recommend the new drug for use if there is convincing evidence that the mean reduction in cholesterol level after one month of use is more than 20 milligrams/deciliter (mg/dl), because a mean reduction of this magnitude would be greater than the mean reduction for the current most widely used drug.

The pharmaceutical company collected data by giving the new drug to a random sample of 50 people from the population of people with high cholesterol. The reduction in cholesterol level after one month of use was recorded for each individual in the sample, resulting in a sample mean reduction and standard deviation of 24 mg/dl and 15 mg/dl, respectively.

- (a) The regulatory agency decides to use an interval estimate for the population mean reduction in cholesterol level for the new drug. Provide this 95 percent confidence interval. Be sure to interpret this interval.
- (b) Because the 95 percent confidence interval includes 20, the regulatory agency is not convinced that the new drug is better than the current best-seller. The pharmaceutical company tested the following hypotheses.

$$H_0: \mu = 20 \text{ versus } H_a: \mu > 20,$$

where μ represents the population mean reduction in cholesterol level for the new drug.

The test procedure resulted in a t -value of 1.89 and a p -value of 0.033. Because the p -value was less than 0.05, the company believes that there is convincing evidence that the mean reduction in cholesterol level for the new drug is more than 20. Explain why the confidence interval and the hypothesis test led to different conclusions.

- (c) The company would like to determine a value L that would allow them to make the following statement.

We are 95 percent confident that the true mean reduction in cholesterol level is greater than L .

A statement of this form is called a one-sided confidence interval. The value of L can be found using the following formula.

$$L = \bar{x} - t^* \frac{s}{\sqrt{n}}$$

This has the same form as the lower endpoint of the confidence interval in part (a), but requires a different critical value, t^* . What value should be used for t^* ?

Recall that the sample mean reduction in cholesterol level and standard deviation are 24 mg/dl and 15 mg/dl, respectively. Compute the value of L .

- (d) If the regulatory agency had used the one-sided confidence interval in part (c) rather than the interval constructed in part (a), would it have reached a different conclusion? Explain.

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Question 3

Qualification: AP Statistics

Areas: Confidence Intervals

Subtopics: Constructing a Confidence Interval , Interpreting a Confidence Interval, Calculating Proportions, Stemplot, One Sample T Interval For A Mean

Paper: Part-A / Series: 2013 / Difficulty: Medium / Question Number: 1

1. An environmental group conducted a study to determine whether crows in a certain region were ingesting food containing unhealthy levels of lead. A biologist classified lead levels greater than 6.0 parts per million (ppm) as unhealthy. The lead levels of a random sample of 23 crows in the region were measured and recorded. The data are shown in the stemplot below.

Lead Levels

2		8
3		0
3		5 8 8
4		1 1 2
4		6 8 8
5		0 1 2 2 3 4
5		9 9
6		3 4
6		6 8

Key: $2|8 = 2.8$ ppm

- (a) What proportion of crows in the sample had lead levels that are classified by the biologist as unhealthy?
- (b) The mean lead level of the 23 crows in the sample was 4.90 ppm and the standard deviation was 1.12 ppm. Construct and interpret a 95 percent confidence interval for the mean lead level of crows in the region.

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Question 4

Qualification: AP Statistics

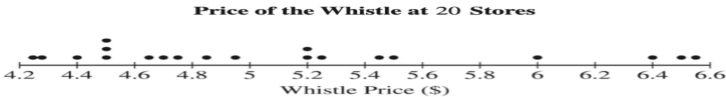
Areas: Confidence Intervals, Data - One Variable

Subtopics: Normality Condition , Pearson's Coefficient of Skewness, One Sample T Interval For A Mean, Summary Statistics/5 Figure Summary, Outliers, Shape of Distribution

Paper: Part-B / Series: 2024 / Difficulty: Somewhat Challenging / Question Number: 6

6. A company sells a certain type of whistle. The price of the whistle varies from store to store. Julio, a statistician at the company, wants to estimate the mean price, in dollars (\$), of this type of whistle at all stores that sell the whistle.
- (a) (i) Identify the appropriate inference procedure for Julio to use.
- (ii) Describe the parameter for the inference procedure you identified in part (a-i) in context.

Julio called the managers of 20 randomly selected stores that sell the whistle and recorded the price of the whistle at each store. Following is a dotplot of Julio's data.



The summary statistics for Julio's data are shown in the following table.

Summary Statistics for Julio's Data							
Sample Size	Mean	Standard Deviation	Minimum	Q_1	Median	Q_3	Maximum
20	5.12	0.743	4.25	4.51	4.885	5.475	6.58

- (b) Julio wants to examine some characteristics of the distribution of the sample of whistle prices.
- (i) Describe the shape of the distribution of the sample of whistle prices. Justify your response using appropriate values from the summary statistics table.
- (ii) Using the $1.5 \times \text{IQR}$ rule, determine whether there are any outliers in the sample of whistle prices. Justify your response.

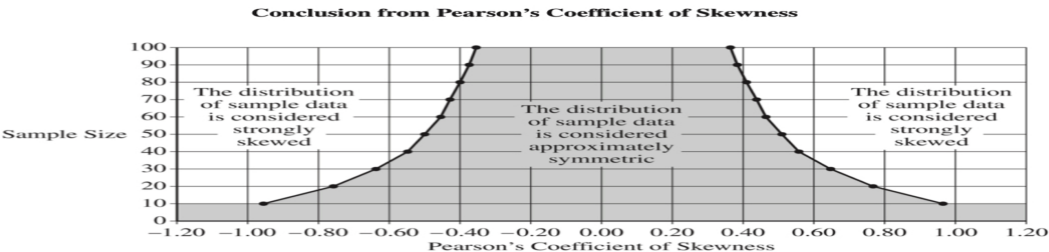
It can often be difficult to determine whether the distribution of sample data is skewed by looking at a graph of the data and the summary statistics, particularly when the sample size is small. Thus, statisticians sometimes measure how skewed a data set is. One such measure is Pearson's coefficient of skewness, which is calculated using the following formula.

$$\text{Pearson's Coefficient of Skewness} = \frac{3(\bar{x} - m)}{s}$$

In the formula, \bar{x} is the sample mean, m is the sample median, and s is the sample standard deviation.

- (c) (i) Calculate Pearson's coefficient of skewness for Julio's sample of 20 whistle prices. Show your work.

The following graph shows conclusions that can be made about the shape of the distribution of sample data based on Pearson's coefficient of skewness and sample size.



- (ii) Indicate the value of the Pearson's coefficient of skewness you calculated in part (c-i) for the appropriate sample size by marking it with an "X" on the preceding graph.
- (d) Consider your work in part (c).
- (i) What should you conclude about the shape of the distribution of the sample of whistle prices? Justify your response.

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Julio's inference procedure in part (a-i) needs one of the following requirements to be satisfied to verify the normality condition.

- The sample size is greater than or equal to 30.
- If the sample size is less than 30, the distribution of the sample data is not strongly skewed and does not have outliers.

Using your response to (d-i) and the preceding requirements, is the normality condition satisfied for Julio? Explain your response.

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